We first note that

$$G = \bigcap_{m\in\mathbb{N}} \bigcup_{n\geq m} \{x \in (0,\infty)|\ nx \in D\}.$$

We notice that there is a countable intersection involved, and so we hope to use Baire's lemma. To this end, we define

$$A_m = \bigcup_{n\geq m} \{x \in (0,\infty)|\ nx \in D\},$$

and note that these sets are open. Indeed if $nx \in D$ then there is an open ball $B_r(nx) \subset D$ and so it follows that $B_{r/n}(x) \subset \{x \in (0,\infty)|\ nx \in D\}$, making A_m the countable union of open sets.

In order to show that each A_m is dense we suppose that it isn't, i.e. if (a,b) is any interval then $(a,b) \cap A_m = \emptyset$. Next we write this intersection as

$$\bigcup_{n\geq m} (na,nb) \cap D = \emptyset,$$

and note that if $m > \frac{a}{b-a}$ then all the intervals in the union overlap and we get $\bigcup_{n\geq m}(na,nb) = (ma,\infty)$. This yields the desired contradiction since D is unbounded.

Being the countable intersection of open and dense sets, G is itself dense by Baire.