So these problems highlight a technique that I found very useful every time I had to pass two limits to deduce a result. The first one is:

Let X be a separable normed vector space and Y a reflexive Banach space, with $(F_n)_{n\in\mathbb{N}} \subset L(X,Y)$ a bounded sequence. Show that there exists a subsequence $\Lambda \subset \mathbb{N}$ such that for any $x \in X$, the sequence $(F_n x)_{n\in\Lambda}$ is weakly convergent in Y.

The second problem came up in a slightly different context last week that really puzzled me and a friend. Suppose you have a strictly convex continuous linear functional $F : X \to \mathbb{R}$ and a sequence $(x_k) \subset X$ weakly convergent to $x \in X$. Then show that $F(x) \leq \liminf_{k\to\infty} F(x_k)$.